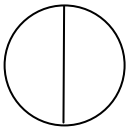
LESSON F-3 IDENTIFYING FRACTIONS

"For each exercise, you may say what the whole is and then say if the picture shows a fraction and why."

"For example, here the whole is a circle and the picture shows a fraction because the circle has been divided into two equal parts; each part is a fraction of the whole. For now we will consider fractions to be equal parts of a whole."

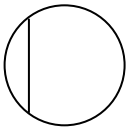


A. "What is the whole?"

B. "Does the picture show a fraction?"

C. "Why?"

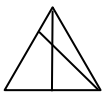
D. "What is the whole?"



E. "Does the picture show a fraction?"

F. "Why?"

G. "What is the whole?"



H. "Does the picture show a fraction?"

I. "Why?"

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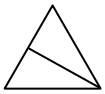
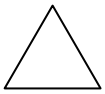
J. “What is the whole?”



K. “Does the picture show a fraction?”

L. “Why?”

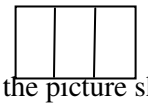
M. “What is the whole?”



N. “Does the picture show a fraction?”

O. “Why?”

P. “What is the whole?”



Q. “Does the picture show a fraction?”

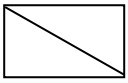
R. “Why?”

S. “What is the whole?”

T. “Does the picture show a fraction?”

U. “Why?”

V. “What is the whole?”



W. “Does the picture show a fraction?”

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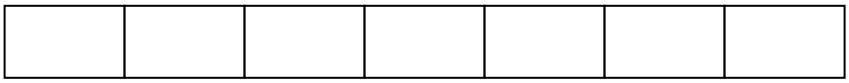
X. “Why?”

Y. “What is the whole?”



Z. “Does the picture show a fraction?”

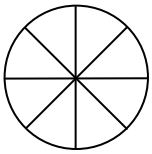
AA. “Why?”

AB. “What is the whole?”

AC. “Does the picture show a fraction?”

AD. “Why?”

AE. “What is the whole?”



AF. “Does the picture show a fraction?”

AG. “Why?”

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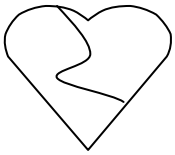
AH. “What is the whole?”

AI. “Does the picture show a fraction?”

AJ. “Why?”

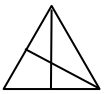
AK. “What is the whole?”



AL. “Does the picture show a fraction?”

AM. “Why?”

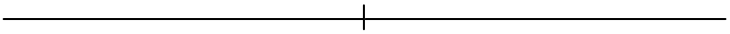
AN. “What is the whole?”



AO. “Does the picture show a fraction?”

AP. “Why?”

AQ. “What is the whole?”



AR. “Does the picture show a fraction?”

AS. “Why?”

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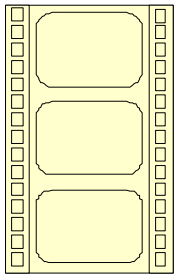
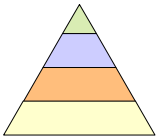
AT. “What is the whole?”



AU. “Does the picture show a fraction?”

AV. “Why?”

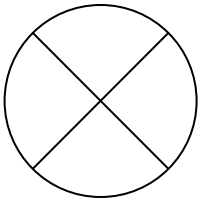
AW. “What is the whole?”



AX. “Does the picture show a fraction?”

AY. “Why?”

AZ. “What is the whole?”



BA. “Does the picture show a fraction?”

BB. “Why?”

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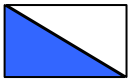
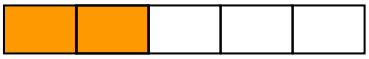
LESSON F-20 NAMING FRACTIONS

"Name the colored part and family for each fraction."

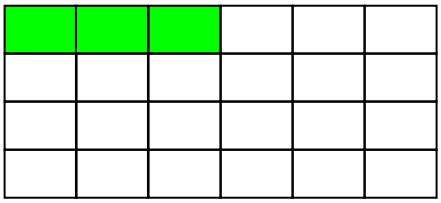
"For example, this is one half because the colored part is one half of the whole:"



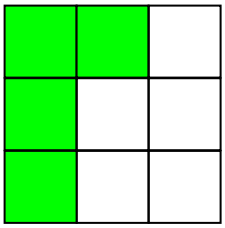
A.



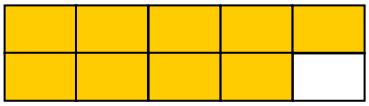
B.



C.



D.



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LESSON F-21 WRITING THE FRACTION

"When we work with members of a fraction family, we identify the number of members with the numerator, and the family name with the denominator. For example, if we have three members of the fifths family, we would write this as:"

$$\frac{3}{5}$$

"This is read as 'three fifths' because there are three members of the fifths family. You may practice writing this fraction."

$\frac{3}{5}$				

A. "How would you write the fraction for one member of the halves family?"

B. "How would you write the fraction for six members of the ninths family?"

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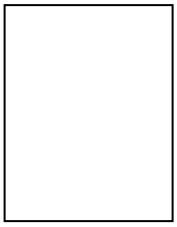
C. “How would you write the fraction for zero members of the tenths family?”



Repeat steps A through C for various members and families of fractions until the student is competent.

D. “Another way to write a fraction is using a diagonal slash:”

/

“For example,

$$\frac{1}{4}$$

“could be written as:”

1/4

“They mean exactly the same thing.”

“You may write each of the following fractions using slashes:”

i. $\frac{3}{4}$

ii. $\frac{5}{8}$

iii. $\frac{8}{5}$

iv. $\frac{2}{10}$

v. $\frac{1}{5000}$

vi. $\frac{10}{71}$

vii. $\frac{1}{n}$

viii. $\frac{n}{k}$

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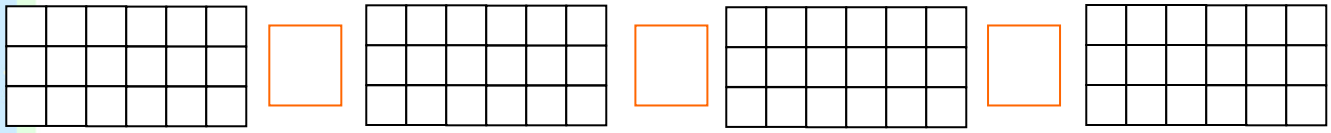
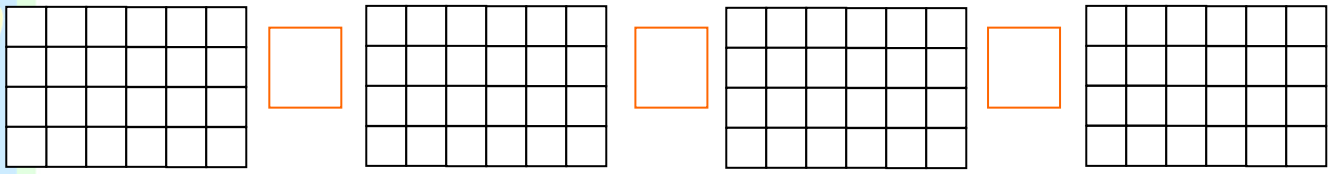
LESSON F-49 FRACTION EQUIVALENCE

"Each rectangle is one unit. Color boxes to match the fraction and put in the correct less-than or greater-than sign inside each red box:"

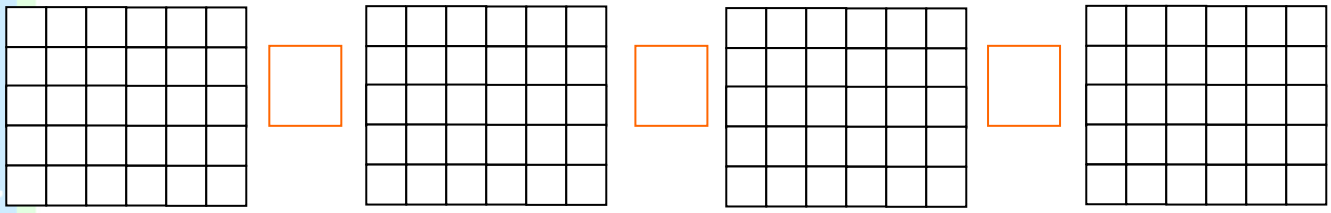
$\frac{1}{18}$ ☐ $\frac{1}{9}$ ☐ $\frac{1}{6}$ ☐ $\frac{1}{3}$



$\frac{1}{2}$ ☐ $\frac{1}{3}$ ☐ $\frac{1}{4}$ ☐ $\frac{1}{6}$



$\frac{1}{5}$ ☐ $\frac{1}{10}$ ☐ $\frac{1}{15}$ ☐ $\frac{1}{30}$



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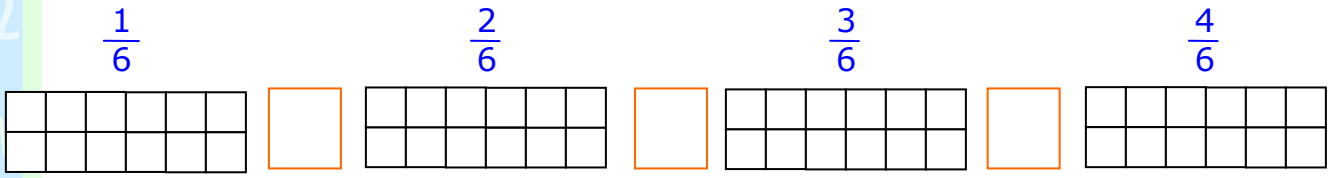
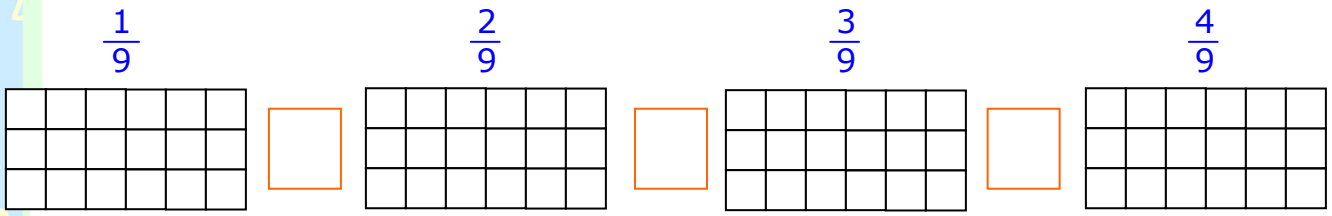
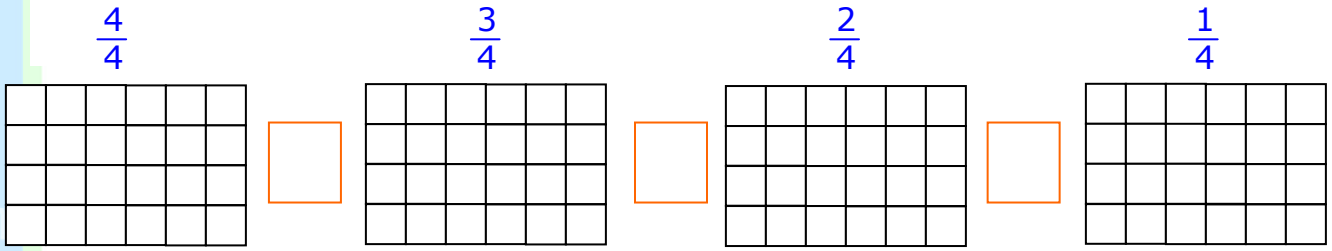
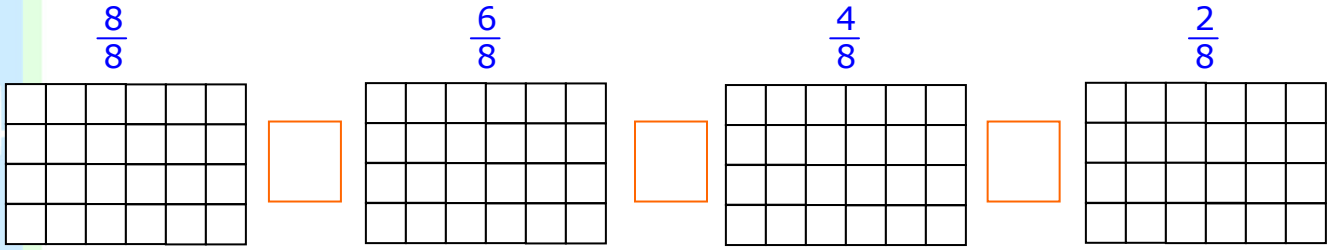
ShillerMath Fractions Activity Book









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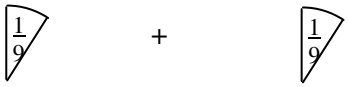
LESSON F-59 FRACTION ADDITION: SAME DENOMINATOR (NINTHS)

A. Put the fraction circles in the work area, in sequence from whole to tenths from left to right. “Here are all the fraction families from wholes to tenths.”

B. Move down the ninths. “Now we will add ninths.”

C. Put one of the ninths to the right and put a plus sign to the right of that.

D. Put the other ninth to the right of the plus sign like this:



While pointing at each of the ninths, “Each of these is a ninth. How many ninths are there?”

E. “One ninth [point to the left ninth] plus [point to the plus sign] one ninth [point to the right ninth]”…

Put an equals sign to the right of the two ninths. “Equals two ninths.” Move the two ninths to the right of the equals sign.

F. Write and position pieces of blank paper as follows:

$\frac{1}{9}$	+	$\frac{1}{9}$	=	$\frac{2}{9}$

Read what’s in the squares from left to right as you point at each square and what’s above it: “One ninth plus one ninth equals two ninths.” Move the two ninths to the right of the equals sign.

“Now you may read what’s in the squares.”

G. Return the ninths and remove the paper squares.

H. Using fraction circles and paper squares for plus and equals, form the following equations one at a time:

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$\frac{1}{9} + \frac{2}{9} = \frac{3}{9}$ $\frac{1}{9} + \frac{3}{9} = \frac{4}{9}$ $\frac{1}{9} + \frac{4}{9} = \frac{5}{9}$

$\frac{1}{9} + \frac{5}{9} = \frac{6}{9}$ $\frac{1}{9} + \frac{6}{9} = \frac{7}{9}$ $\frac{1}{9} + \frac{7}{9} = \frac{8}{9}$

$\frac{1}{9} + \frac{8}{9} = 1$

For each equation, "You may say the words for this equation. Would you like to write the equation on paper?" Let the student do so if amenable. Otherwise you may write it.

I. "You may write what you need to make the following equations correct:"

$\frac{0}{9} + \frac{1}{9} =$ $\frac{0}{9} + \frac{2}{9} =$ $\frac{0}{9} + \frac{3}{9} =$

$\frac{0}{9} + \frac{4}{9} =$ $\frac{0}{9} + \frac{5}{9} =$ $\frac{0}{9} + \frac{6}{9} =$

$\frac{0}{9} + \frac{7}{9} =$ $\frac{0}{9} + \frac{8}{9} =$ $\frac{0}{9} + \frac{9}{9} =$

$\frac{1}{9} + \frac{2}{9} =$ $\frac{1}{9} + \frac{3}{9} =$ $\frac{1}{9} + \frac{4}{9} =$

$\frac{1}{9} + \frac{5}{9} =$ $\frac{1}{9} + \frac{6}{9} =$ $\frac{1}{9} + \frac{7}{9} =$

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$\frac{1}{9} + \frac{8}{9} =$

$\frac{2}{9} + \frac{3}{9} =$

$\frac{2}{9} + \frac{4}{9} =$

$\frac{2}{9} + \frac{5}{9} =$

$\frac{2}{9} + \frac{6}{9} =$

$\frac{2}{9} + \frac{7}{9} =$

$\frac{3}{9} + \frac{4}{9} =$

$\frac{4}{9} + \frac{5}{9} =$

$\frac{2}{9} + \frac{2}{9} =$

$\frac{3}{9} + \frac{3}{9} =$

$\frac{4}{9} + \frac{4}{9} =$

$\frac{5}{9} + \frac{4}{9} =$

$\frac{0}{9} + \frac{1}{9} = \frac{1}{9}$

$\frac{1}{9} + \frac{7}{9} = \frac{8}{9}$

$\frac{0}{9} + \frac{6}{9} = \frac{6}{9}$

$\frac{1}{9} + \frac{2}{9} = \frac{3}{9}$

$\frac{3}{9} + \frac{2}{9} = \frac{5}{9}$

$\frac{1}{9} + \frac{1}{9} = \frac{2}{9}$

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LESSON F-100 FRACTIONS OF AN HOUR

"One hour equals 60 minutes:"

$$1 \text{ h} = 60 \text{ m}$$

"For each of the following you may fill in the blanks and write the equation. *A* was done for you:"

A. three quarters of an hour = _45_ minutes $\frac{3}{4} \times 60 = \frac{180}{4} = 45$

B. two thirds of an hour = ____ minutes

C. four thirds of an hour = ____ minutes

D. three fifths of an hour = ____ minutes

E. five sixths of an hour = ____ minutes

F. three tenths of an hour = ____ minutes

G. five tenths of an hour = ____ minutes

H. seven tenths of an hour = ____ minutes

I. nine tenths of an hour = ____ minutes

J. one twelfth of an hour = ____ minutes

K. five twelfths of an hour = ____ minutes

L. seven twelfths of an hour = ____ minutes

M. eleven twelfths of an hour = ____ minutes

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LESSON F-130 FRACTION SEQUENCES

"Write in the next three terms and the rule for each of these sequences:"

A. $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ _____ _____ _____

Rule:

B. $\frac{22}{7}$ $\frac{19}{7}$ $\frac{16}{7}$ $\frac{13}{7}$ _____ _____ _____

Rule:

C. $\frac{1}{13}$ $\frac{2}{13}$ $\frac{4}{13}$ $\frac{8}{13}$ _____ _____ _____

Rule:

D. $\frac{1}{5}$ $\frac{1}{3}$ $\frac{3}{7}$ $\frac{1}{2}$ _____ _____ _____

Rule:

Hint: First change some of the fractions to make a pattern of the denominators.

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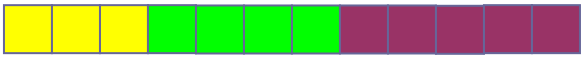
LESSON F-132 FRACTION STRIPS

"This is a fraction strip, with twelve boxes:"



A. "You may write an equation to show how three twelfths plus four twelfths plus five twelfths equals one."

B. "Now rewrite the equation from step A with reduced fractions."

C. "Write a new equation to show how much the yellow and purple boxes add up to."

D. "Now rewrite the equation from step C with reduced fractions."

E. "Write a new equation that shows how much is left from the fraction strip after taking away the yellow boxes."

F. "Now rewrite the equation from step E with reduced fractions."

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LESSON F-140 LEAST COMMON DENOMINATOR USING PRIME FACTORS

Note: Please do the Book 6 lessons on prime factors prior to this lesson.

A. "When adding fractions we have seen that the two fractions need to be from the same family."

"We can change a fraction's family by multiplying its numerator and denominator by the same integer."

"When adding two fractions, for example:"

$$\frac{3}{4} + \frac{2}{5}$$

"it makes sense to multiply the numerator and denominator of the fraction three fourths by five and to multiply the numerator and denominator of the fraction two fifths by four because that will put both fractions into the twentieths fraction family:"

$$\frac{15}{20} + \frac{8}{20}$$

"and because both fractions are now in the same fraction family we can add them together:"

$$\frac{15}{20} + \frac{8}{20} = \frac{23}{20}$$

"The number twenty is the smallest number that is a multiple of both four and five. It is called the least common denominator."

"What is the least common denominator in this example? Why?"

B. "Another way to find the least common denominator is to find out all the prime factors for each denominator. For example, with the above example, adding three fourths to two fifths, we start with the prime factors for the two denominators four and five:"

$$4 = 2 \times 2$$

$$5 = 5$$

"The smallest number that will have both of those prime factors is:"

$$20 = 2 \times 2 \times 5$$

"Let's see, we've got the prime factor two twice from four [point] and the prime factor five from five [point]. Good."

"You may show me where the prime factors in the LCD come from."

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C. “Let’s try again with denominators 6 and 15:”

$$6 = 2 \times 3$$

$$15 = 3 \times 5$$

“The least common denominator (we’ll call this LCD from now on) has all the prime factors, so it is:”

$$LCD = 2 \times 3 \times 5$$

“which is 30. Let’s just be sure: Our LCD has one two, which comes from the six [point], and one three, which happens to come from both the six and the fifteen [point at both 3s] but we only need one three since no denominator has more than one three, and the five from the fifteen [point]. Good.”

“You may show me where the prime factors in the LCD come from.”

D. “Let’s try again with denominators of 6 and 45:”

$$6 = 2 \times 3$$

$$45 = 3 \times 3 \times 5$$

“The LCD is:”

$$LCD = 2 \times 3 \times 3 \times 5$$

“which is 90. Note how we needed two 3s in the LCD because 45 has two 3s in its prime factors.”

“You may show me where the prime factors in the LCD come from.”

E. “Using prime factors, determine the LCD for the following. Show the prime factors of each number and the prime factors for the LCD:”

i. 3, 5

ii. 4, 6

iii. 3, 2

iv. 2, 3

v. 1,000, 500

vi. 110, 1210

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LESSON F-155 MULTIPLYING FRACTIONS: CONCRETE

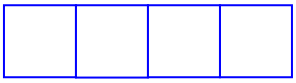
"We have done addition and subtraction of fractions; now let's multiply fractions."

A. "For example, let's say we wanted to take one fourth of two thirds:"

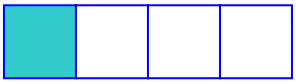
$$\frac{1}{4} \times \frac{2}{3}$$

"Let's try to do this using rectangles."

"The first fraction is one fourth, so it is in the fourths family. Let's start with a 1-by-4 rectangle to represent the fourths family:"



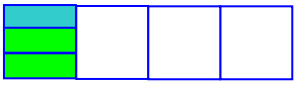
"Now we want to show one fourth. The aqua box represents one fourth of the whole:"



"This is one fourth."

"Now we want two thirds of the aqua box."

"So let's divide the aqua box into thirds and color two of those three parts green:"



i. "True or false: The green parts represent the product of one fourth and two thirds."

"But hmmm, we've got uneven parts. Each of the three little colored parts is one third the size of each of the white boxes. It would be nice to have the white boxes divided into thirds, too, so all the smallest parts are the same size:"

"There we go! Now every part is a part of the same family."

ii. "How many individual parts are there now?"

iii. "What is the fraction family?"

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iv. “How many are colored green?”

v. “What fraction of the area is the product of one fourth and two thirds?”

vi. “By looking at the number of the fractions being multiplied, say how you could figure out the fraction family of the product.”

B. Using ShillerMath Graphsheets, repeat step A for the following fraction multiplications:

Fraction 1	×	Fraction 2	Green parts	Total parts	Product
$\frac{2}{5}$	×	$\frac{1}{2}$			
$\frac{2}{3}$	×	$\frac{1}{4}$			
$\frac{1}{3}$	×	$\frac{3}{4}$			
$\frac{5}{6}$	×	$\frac{1}{2}$			

C. “For which of the products in step B could we have used a rectangle with a fewer number of parts and how many parts would that be?”

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