LESSON 4-5 THE LAW OF COMMUTATIVITY

"Axioms [AXE – ee – ums] are things we assume to be true because they seem obvious but we cannot prove them. Say with me: *axiom*."

A. "For example, if three plus four is seven, what is four plus three?"

$$3 + 4 = 7$$

$$4 + 3 =$$

B. "True or false:"

$$3 + 4 = 4 + 3$$

C. "True or false:"

$$a + b = b + a$$

D. "When people commute to work, they go back and forth between home and work. The law of commutativity for addition says that when you add two numbers together it doesn't matter which you add first. Say with me: *law of commutativity*."

"The law of commutativity is an axiom. We cannot prove it but we know it is true."

E. "In step C we saw that the law of commutativity holds for addition."

F. "Let's see if the law of commutativity holds for multiplication. True or false:"

$$3 \times 4 = 4 \times 3$$

G. "True or false:"

$$a \times b = b \times a$$

H. "Does the law of commutativity hold for multiplication?"

I. "Let's see if the law of commutativity holds for subtraction. True or false:"

$$3 - 4 = 4 - 3$$

J. “True or false:”

$$a - b = b - a$$

K. “True or false: The law of commutativity holds for subtraction.”

L. “Let’s see if the law of commutativity holds for division. True or false:”

$$3 \div 4 = 4 \div 3$$

M. “True or false:”

$$a \div b = b \div a$$

N. “True or false: The law of commutativity holds for division.”

LESSON 4-33 PERCENT ETYMOLOGY AND SYMBOL

"The word *percent* comes from Latin:"

per cent

"[point] *per* means 'divided by'" "[point] *cent* means 'hundred'"

"So percent means divided by one hundred."

"100 percent, which is all of something, is 100 divided by 100, or 1."

"50 percent, which is half of something, is 50 divided by 100, or one half."

A. "True or false:"

i. "A percentage is something divided by 50."

ii. "A percentage can be thought of as a division problem."

B. "50 percent could be written as:"

$$50 \div 100$$

"Every division can be written as a fraction. 50 percent is the same as:"

$$\frac{50}{100}$$

"Since there are two fifties in one hundred, 50 percent is the same as one half."

C. "True or false:"

i. "A percentage can always be written as a fraction."

ii. "If 50 percent is the same as one half, 25 percent is the same as one-fifth."

iii. "If 100 percent is the same as 1, then 200 percent is the same as 2."

iv. "If 50 percent is the same as one half, how many percent would one quarter be?"

D. "Percent has an abbreviation. This is the percent sign:"

%

"Show me the percent sign in this list of symbols:"

! @ # $ % ^ & * ( ) _ + { } | < > ?

E. "What is this symbol called?"

%

"Whenever we see a percent sign, we say the word *percent*. This is 50%:"

50%

F. "Say out loud each of the following:"

i. 100%

ii. 200%

iii. 0%

iv. 25%

v. 75%

vi. 1/2%

vii. 1,000%

LESSON 4-61 OPERATIONS INTRODUCTION

"An operation is something we do to numbers."

"For example, addition is an operation that adds two numbers together. The plus sign is the operator."

$$3 + 6$$

"Three plus six. Here the operation is addition and the operator is the plus sign."

A. "Tell me the operation and show me the operator in each of the following:"

3×6

$1 + 3$

$1{,}039{,}012 \div 66{,}372$

23×3

$0 - 3{,}000$

$2 + 41$

$5 \div 23{,}920$

"All these operations are called *binary* operations because they all involve two numbers. Operations that involve exactly two numbers are called binary operations. Binary operations."

"Some expressions can have more than one operation. For example, this expression has two operations: addition and division:"

$10 + 12 \div 6$

B. "Tell me how many operations each of the following expressions has and name the operators:"

$3 \times 6 + 2$

$1 + 3 \times 0 + 13$

$10 \times 6 + 33 \div 11$

$23 - 3 \times 2 + 30 \div 1$

$0 \times 3000 + 10 \div 2 - 5$

$2 \div 41 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

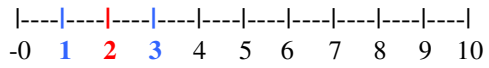
$100 + 50 \times 2 + 300 + 200 + 100$

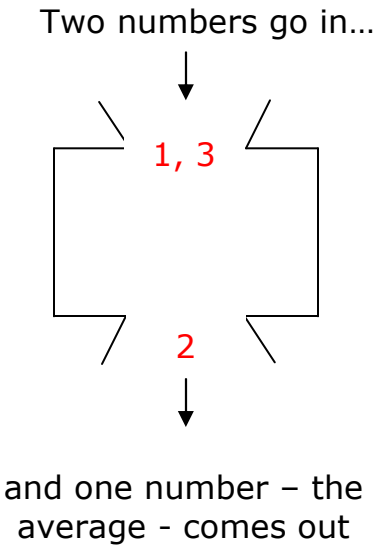
6

LESSON 4-112 AVERAGE

"The average of two numbers is the number exactly in between the two numbers."

"For example on the number line, 1 and 3 are shown in blue. The average of 1 and 3 is 2 because 2 is right in the middle between 1 and 3:"





A. "What is the average of 1 and 3?"

B. "What is the average of 3 and 1?"

C. "If you average two numbers, does it matter which you say first?"

"You may figure out the average for the following sets of numbers:"

D. 0, 2

E. 0, 4

F. 0, 6

G. 0, 8

H. 0, 10

I. zero and anything

J. anything and zero

K. 1, 5

L. 2, 6

M. 1, 1

N. 2, 2

O. 3, 3

P. any number and itself

Q. "Can the average be less than both numbers? Why?"

R. "Can the average be greater than both numbers? Why?"

S. "When is the average the same as one or both of the numbers? Why"

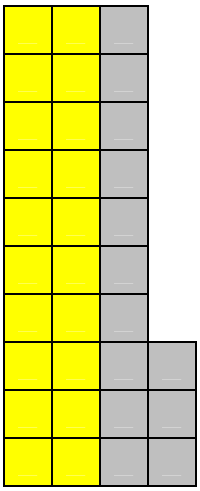
T. "Can you say a rule for coming up with the average of two numbers?"

If the student has difficulty, keep giving examples (for now do not give a pair of numbers where one is even and one is odd) until a pattern emerges in the student's mind.

LESSON 4-135 VISUAL ADDITION AND SUBTRACTION

"Look at these pictures and write an addition and subtraction equation about each. For example, for picture A:"

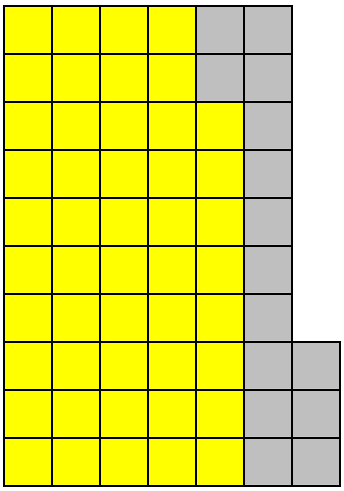
A.



"You could write all these equations:"

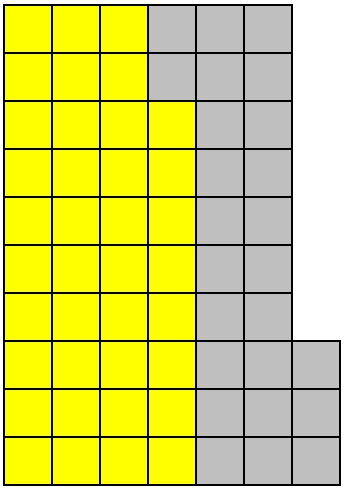
20 + 10 + 3 = 33
20 + 13 = 33
(10 + 10) + (10 + 3) = 33
33 – 20 = 13
33 – 13 = 20
33 – 3 – 10 = 20

B.



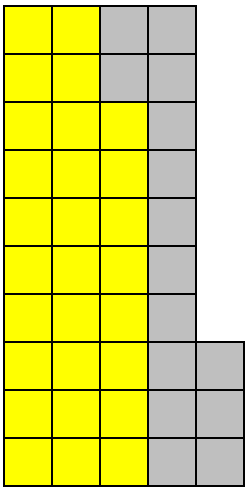
Equations for picture B:

C.



Equations for picture C:

D.



Equations for picture D:

LESSON 4-140 DECIMAL NUMBERS: THOUSANDTHS

Take out ten thousandth squares (with the red print) and one hundredth square.

Hold up the hundredth square. “This is one hundredth or zero point zero one.”

Show ten of the one thousandths squares with the 0.001 side facing up. “A hundredth may be divided into ten thousandths.”

A. Hold up one thousandth square showing the 1/1,000 side facing up. “This is one thousandth.”

B. Turn it to show the 0.001 side. “This is also one thousandth. Zero point zero zero one. Say with me: *zero point zero zero one*.”

C. Put all the material back down in the work area. “Show me one thousandth.”

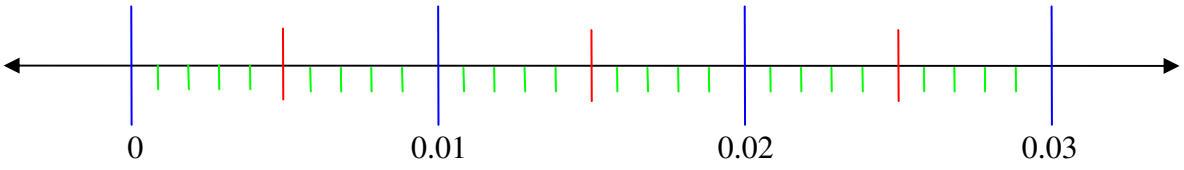
D. “Show me zero point zero zero one.”

E. Hold up a thousandth. “What is this called?”

F. Keep holding up the thousandth. “What is another name for this?”

G. “Which is larger: One hundredth or one thousandth?”

H. “Show me the tick for one thousandth on this number line and write zero point zero zero one under the tick:”



I. Hold up two thousandth squares showing the 1/1,000 sides facing up. “One thousandth plus one thousandth is two thousandths. These are two thousandths.”

J. Turn them to the 0.001 side. “These are also two thousandths. [Point at each square as you say the following:] Zero point zero zero one plus zero point zero zero one is zero point zero zero two. Say with me: *zero point zero zero two*.”

K. Put all the thousandth squares back down in the work area. “Show me two thousandths.”

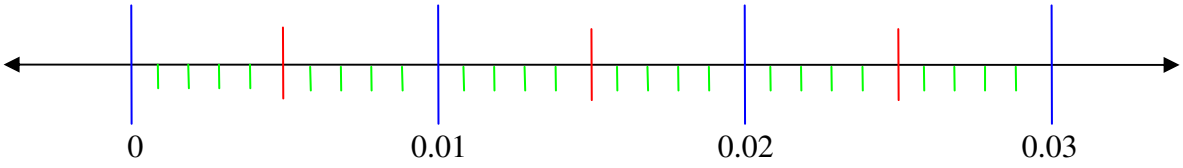
L. “Show me zero point zero zero two.”

M. Hold up two thousandths. “What is this called?”

N. Keep holding up the thousandths. “What is another name for this?”

O. “Which is smaller: Two hundredths or two thousandths?”

P. “Show me the tick for two thousandths on this number line and write zero point zero zero two under the tick:”



Repeat steps I through P for three thousandths through ten thousandths until the student is competent.